Samples Problems

Indefinite Integrals (Section 4.1)

1. $\int \frac{sinx}{cos^2x} dx$

2. $\int \frac{x+1}{\sqrt{x}} dx$

3. $\int \frac{1}{x^4} dx$

Definite Integrals (Section 4.3)

4. $\int_1^5 x^2 dx$

5. $\int_0^1 (2t-1)^2 dt$

6. $\int_1^5 (-x^2+4x-3) dx$

7. $\int_{-\pi/6}^{\pi/6} sec^2x \, dx$

U-substitution (Section 4.5)

8. $\int sin^2x(3x)\, cos(3x)\, dx$

9. $\int 3x^2\sqrt{x^3-2}\, dx$

10. $\int x^2(x^3+5)^6 \, dx$

11. $\int \frac{x}{\sqrt{2x+1}} dx$

GROUP 4 Sample Problems

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Fundamental Theorem of Calculus.

1 Evaluate the definite integral of the algebraic function.

a) $\int_0^3 |x^2-4|\,dx$ b) $\int_0^\pi (1+\sin x)\,dx$ c) $\int_{-1}^1 (\sqrt[3]{t}-2)\,dt$ d) $\int_1^8 \sqrt{\frac{2}{x}}\,dx$

2 Find the area of the region bounded by the graphs of the equations:

a) $y=3x^2+1,\ x=0,\ x=2,\ y=0$

b) $y=3x^2-8x+7,\ x=0,\ x=3,\ y=0$

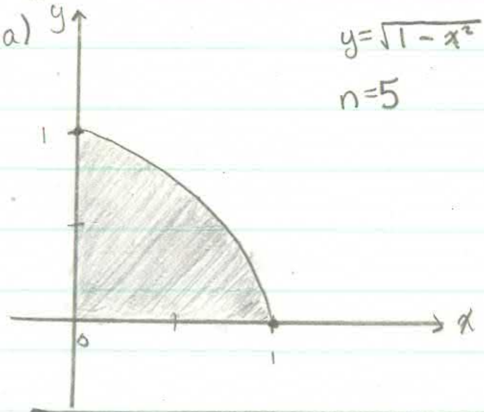
3 Find F as a function of x and evaluate it at $x=3,\ x=5,\ x=9$

a) $F(x)=\int_0^x (t-5)\,dt$ b) $F(x)=\int_1^x \frac{10}{v^2}\,dv$ c) $F(x)=\int_1^x \cos\theta\,d\theta$

Numerical Approximations to Definite Integrals

1 Use upper and lower sums to approximate the area of the region using the given number of subintervals (of equal width).

a) $y=\sqrt{1-x^2}$, $n=5$



b) $y=\sqrt{x}+2$ on closed interval $[0,2]$, $n=8$

2

t (seconds)	0	10	20	30	40	50	60	70	80
v(t) (feet per sec)	6	13	21	29	35	46	53	58	64

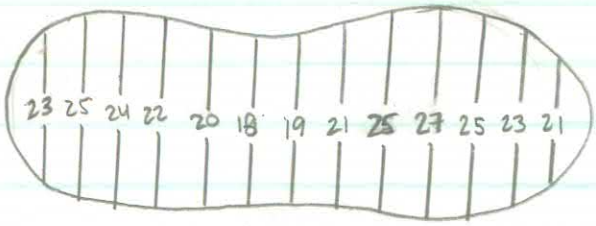
A Rocket Z has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t=0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

3 Approximate the definite integral using the Trapezoidal Rule with $n=4$.

a) $\int_0^2 \sqrt{1+x^3}\,dx$ b) $\int_1^{1.1} \sin x^2\,dx$

4 Approximate the area of the golf course using the Trapezoidal Rule.

The distance between each measured cross-sections is 10 meters.



Group 4

Practice problems:

1. Find the upper and lower Riemann sums for the graphs:

   a) $y=\sqrt{x}$ [0,1] 4 rectangles

   b) $y=\dfrac{1}{x}$ [1,2] 5 rectangles

   c) $y=\dfrac{10}{x^2+1}$ [0,2] 4 rectangles

2. Use the Fundamental Theorem of Calculus to solve the definite integral.

   a) $\int_0^4 (2+x)dx$

   b) $\int_{-1}^1 (4x^3-2x)dx$

   c) $\int_4^9 (x\sqrt{x})dx$

   d) $\int_0^{\frac{3\pi}{4}} (\sin\theta)d\theta$

3. Find the area under the curve.

   a) $\int_1^3 (2x-1)dx$

   b) $\int_0^1 (x-x^3)dx$

4. Use the Rules for definite integrals to evaluate.

   A) Given that $\int_0^5 f(x)dx=10$ and $\int_5^7 f(x)dx=3$ evaluate:

   a) $\int_0^7 f(x)dx$

   b) $\int_5^0 f(x)dx$

   c) $\int_5^5 f(x)dx$

   d) $\int_0^5 3f(x)dx$

   B) Given that $\int_2^6 f(x)dx=10$ and $\int_2^6 g(x)dx=-2$ evaluate:

   a) $\int_2^6 f(x)+g(x)dx$

   b) $\int_2^6 g(x)-f(x)dx$

   c) $\int_2^6 2g(x)dx$

   d) $\int_2^6 3f(x)dx$

Answer Key

1. $\int \frac{sinx}{cos^2x}dx = \int \left(\frac{sin\,x}{cosx}\right)\left(\frac{1}{cosx}\right)dx = \int (secxtanx)dx = \sec(x) + c$

2. $\int \frac{x+1}{\sqrt{x}}dx = \int \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right)dx = \int \left(x^{\frac{1}{2}} + x^{\frac{-1}{2}}\right)dx = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

3. $\int \frac{1}{x^4}dx = \int (x^{-4}) = -\frac{1}{3}x^{-3} + c = -\frac{1}{3x^3} + c$

4. $\int_1^5 x^2\,dx = \frac{x^3}{3} \begin{pmatrix}5\\1\end{pmatrix} = \frac{125}{3} - \frac{1}{3} = \frac{124}{3}$

5. $\int_0^1 (2t-1)^2\,dt = \int_0^1 (4t^2 - 4t + 1)dt = \left(\frac{4}{3}t^3 - 2t^2 + t\right)\begin{pmatrix}1\\0\end{pmatrix} = \frac{4}{3} + 2 + 1 = \frac{1}{3}$

6. $\int_1^5 (-x^2 + 4x - 3)\,dx = -\frac{1}{3}x^3 + 2x^2 - 3x\begin{pmatrix}5\\1\end{pmatrix} = (-\frac{1}{3}(5)^3 + 2(5)^2 - 3(5)) - (-\frac{1}{3}(1)^3 + 2(1)^2 - 3(1))$

$= 0 - \left(-\frac{4}{3}\right) = \frac{4}{3}$

7. $\int_{-\pi/6}^{\pi/6} sec^2x\,dx = tanx \begin{pmatrix}\pi/6\\-\pi/6\end{pmatrix} = tan\left(-\frac{\pi}{6}\right) - tan\left(\frac{\pi}{6}\right) = \left(-\frac{1}{\sqrt{3}}\right) - \left(\frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$

8. $\int sin^2(3x)cos(3x)dx$ $\qquad u = sin(3x) \qquad du = 3cos(3x)dx$

↑ ↑

U du

$\frac{1}{3}\int u^2\,du = \frac{1}{3} \times \frac{u^3}{3} + c = \frac{1}{9}sin^3(3x) + c$

9. $\int 3x^2\sqrt{x^3 - 2}\,dx$ $\qquad u = x^3 - 2 \qquad du = 3x^2\,dx$

↑ ↑

u du

$$\int \sqrt{u}\, du = \int u^{1/2} = \frac{2}{3}u^{3/2} + c = \frac{2}{3}(x^3 - 2)^{\frac{3}{2}} + c$$

10. $\int x^2 (x^3 + 5)^6\, dx$ $\qquad u = x^3 + 5 \qquad du = 3x^2\, dx$

↑ ↑

Du u

$$\frac{1}{3}\int du\, (u)^6\, dx = \frac{1}{3} * \frac{1}{7}(u)^7 = \frac{1}{21}(x^3 + 5)^7 + c$$

11. $\int_0^4 \frac{x}{\sqrt{2x+1}}$ $\qquad u = 2x + 1 \qquad du = 2\, dx$

$$x = \frac{1}{2}(u - 1) \qquad dx = \frac{1}{2}du$$

$$\int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\, \frac{1}{2}du = \frac{1}{4}\int \frac{u-1}{\sqrt{u}}\, du = \frac{1}{4}\int (u-1)(u)^{\frac{-1}{2}}\, du =$$

$$\frac{1}{4}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du = \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) = \frac{1}{6}(2x+1)^{\frac{3}{2}} - \frac{1}{2}(2x+1)^{\frac{1}{2}} \binom{4}{0}$$

$$\left(\frac{1}{6}(27) - \frac{3}{2}\right) - \left(\frac{1}{6} - \frac{1}{2}\right) = \frac{10}{3}$$

Jessica Tjong

GROUP 4 ANSWER KEY

Fundamental Theorem of Calculus

1. a) $\int_0^3 |x^2-4|\,dx = \int_0^2 -(x^2-4)\,dx + \int_2^3 (x^2-4)\,dx$

$= \left[\frac{-x^3}{3}+4x\right]_0^2 + \left[\frac{x^3}{3}-4x\right]_2^3$

$= -\frac{8}{3}+8+\left[\frac{27}{3}-12-\left(\frac{8}{3}-8\right)\right]$

$= -\frac{8}{3}+8+\left(5-\frac{8}{3}\right)$

$= -\frac{16}{3}+13$

$= \frac{23}{3}$

b) $\int_0^\pi (1+\sin x)\,dx = x-\cos x\Big]_0^\pi$

$= (\pi-\cos\pi)-(0-\cos(0))$

$= (\pi-(-1))-(-1)$

$= \pi+1+1$

$= \pi+2$

c) $\int_{-1}^{1} (\sqrt[3]{t}+2)\,dt = \frac{3}{4}t^{4/3}-2t\Big]_{-1}^{1}$

$= \frac{3}{4}(1)^{4/3}-2(1)-\left[\frac{3}{4}(-1)^{4/3}-2(-1)\right]$

$= \frac{3}{4}-2+\frac{3}{4}-2$

$= -4$

d) $\int_1^8 \sqrt{\frac{2}{x}}\,dx = \int_1^8 (\sqrt{2}\,x^{-1/2})\,dx$

$= \sqrt{2}\,(2x^{1/2})\Big]_1^8$

$= \sqrt{2}(2\sqrt{8})-\sqrt{2}(2\sqrt{1})$

$= 8-2\sqrt{2}$

$\doteq 5.172$

2. a) $\int_0^2 (3x^2+1) = x^3+x\Big]_0^2$

$= 8+2$

$= 10$

b) $\int_0^3 (3x^2-8x+7) = x^3-4x^2+7x\Big]_0^3$

$= 27-4(9)+21$

$= 12$

3. a) $F(x) = \int_0^x (t-5)\,dt$

$F(x) = \frac{t^2}{2}-5t\Big]_0^x$

$= \frac{x^2}{2}-5x$

$F(3) = \frac{9}{2}-15$

$= \frac{-21}{2}$

$F(5) = \frac{25}{2}-25$

$= \frac{-25}{2}$

$F(9) = \frac{81}{2}-45$

$= \frac{-9}{2}$

b) $F(x) = \int_1^x \frac{10}{v^2}\,dv$

$= 10\int_1^x v^{-2}\,dv$

$= 10\left(\frac{-1}{v}\right)\Big]_1^x$

$= \frac{-10}{x}-\left[10\left(\frac{-1}{1}\right)\right]$

$= -\frac{10}{x}+10$

$F(3) = \frac{-10}{3}+10$

$= \frac{20}{3}$

$F(5) = \frac{-10}{5}+10$

$= 8$

$F(9) = \frac{-10}{9}+10$

$= \frac{80}{9}$

3 c) $F(x) = \int_1^x \cos\theta \, d\theta$

$= \sin\theta \Big]_1^x$

$= \sin x - \sin(1)$

$F(3) = \sin 3 - \sin 1 \approx -0.700$

$F(5) = \sin 5 - \sin 1 \approx -1.800$

$F(9) = \sin 9 - \sin 1 \approx -0.429$

<u>Numerical Approximations to Definite Integrals</u>

1 a) $y = \sqrt{1-x^2}$ $h = \frac{b-a}{n} = \frac{1-0}{5}$

upper: $A \approx \frac{1}{5}\left(\sqrt{1-0^2} + \sqrt{1-(\frac{1}{5})^2} + \sqrt{1-(\frac{2}{5})^2} + \sqrt{1-(\frac{3}{5})^2} + \sqrt{1-(\frac{4}{5})^2}\right)$

$\boxed{\doteq 0.859}$

lower: $A \approx \frac{1}{5}\left(\sqrt{1-(\frac{1}{5})^2} + \sqrt{1-(\frac{2}{5})^2} + \sqrt{1-(\frac{3}{5})^2} + \sqrt{1-(\frac{4}{5})^2} + \sqrt{1-1^2}\right)$

$\boxed{\doteq 0.659}$

b) $y = \sqrt{x} + 2$ $h = \frac{2-0}{8}$

lower: $A \approx \frac{1}{4}\left(\sqrt{0}+2 + \sqrt{\frac{1}{4}}+2 + \sqrt{\frac{1}{2}}+2 + \sqrt{\frac{3}{4}}+2 + \sqrt{1}+2 + \sqrt{\frac{5}{4}}+2 + \sqrt{\frac{3}{2}}+2 + \sqrt{\frac{7}{4}}+2\right)$

$\boxed{\doteq 5.685}$

upper: $A \approx \frac{1}{4}\left(\sqrt{\frac{1}{4}}+2 + \sqrt{\frac{1}{2}}+2 + \sqrt{\frac{3}{4}}+2 + \sqrt{1}+2 + \sqrt{\frac{5}{4}}+2 + \sqrt{\frac{3}{2}}+2 + \sqrt{\frac{7}{4}}+2 + \sqrt{2}+2\right)$

$\boxed{\doteq 6.038}$

2 $\int_{10}^{70} v(t)\,dt = \frac{70-10}{3}(21+35+53)$

$= 20(109)$

$\boxed{= 2180}$

3 a) $\int_0^2 \sqrt{1+x^3}\,dx$ $h = \frac{2-0}{4} = \frac{1}{2}$

$\int_0^2 \sqrt{1+x^3}\,dx = \frac{1}{2(2)}\left(\sqrt{1+(0)^3} + 2\sqrt{1+(\frac{1}{2})^3} + 2\sqrt{1+(1)^3} + 2\sqrt{1+(\frac{3}{2})^3} + \sqrt{1+(2)^3}\right)$

$\boxed{\doteq 3.283}$

b) $\int_1^{1.1} \sin x^2\,dx$ $h = \frac{1.1-1}{4} = 0.025$

$\int_1^{1.1} \sin x^2\,dx = \frac{0.025}{2}\left(\sin(1)^2 + 2\sin(1.025)^2 + 2\sin(1.05)^2 + 2\sin(1.075)^2 + \sin(1.1)^2\right)$

$\boxed{\doteq 0.0891}$

4 $n = 14$ $A \approx \frac{140}{2(14)}\big(0 + 2(23) + 2(25) + 2(24) + 2(22) + 2(20) + 2(18) + 2(19) + 2(21) + 2(25) +$
$2(27) + 2(25) + 2(23) + 2(21) + 0\big)$

$= 5(586)$

$\boxed{= 2930 \text{ m}^2}$

GROUP 1

Answer Key

1. a) $A(u) = \frac{1}{4}((.5) + (.707) + (.866) + (1))$

$A(l) = \frac{1}{4}((0) + (.5) + (.707) + (.866))$

U= .768 L= .518

b) $A(l) = \frac{1}{5}((.833) + (.714) + (.625) + (.556) + (.5))$

$A(u) = \frac{1}{5}((1) + (.833) + (.714) + (.625) + (.556))$

U= .746 L= .646

c) $A(l) = \frac{1}{2}((8) + (5) + (3.077) + (2))$

$A(u) = \frac{1}{2}((10) + (8) + (5) + (3.077))$

U= 13.038 L= 9.038

2 a) $\int_0^4 (2 + x)dx = 2x + \frac{1}{2}x^2 \Big|_0^4 = 16 - 0 = 16$

b) $\int_{-1}^1 (4x^3 - 2x)dx = x^4 - x^2 \Big|_{-1}^1 = 0 - 0 = 0$

c) $\int_4^9 (x\sqrt{x})dx = \frac{2}{5}x^{\frac{5}{2}} \Big|_4^9 = 98.2 - 12.8 = \frac{422}{6}$

d) $\int_0^{\frac{3\pi}{4}} (\sin\theta)d\theta = \cos\theta \Big|_0^{\frac{3\pi}{4}} = 1 + \frac{\sqrt{2}}{2} = \frac{(\sqrt{2}+2)}{2}$

3. a) $\int_1^3 (2x - 1)dx = x^2 - x \Big|_1^3 = 6 - 0 = 6$

b) $\int_0^1 (x - x^3)dx = \frac{1}{2}x^2 - \frac{1}{4}x^4 = .25 - 0 = \frac{1}{4}$

4. A) a)10+3=13 b)-10 c)0 d)3(10)=30

B) a)10+(-2)=8 b)(-2)-10=-12 c)2(-2)=-4 d)3(10)=30